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AVERAGE AND PROBABILITY.

167. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

A line l is divided into n segments by $n-1$ points taken at random on it; find the mean value of the product of p of the segments, the p segments being taken at random and p being less than n .

Solution by the PROPOSER.

Let the p segments taken at random in one instance be $a, b, c, \dots$. If p other points are taken at random on the line, the chance that the first will fall on a is a/l ; that the second will fall on b is b/l ; and so on. Hence, the chance that the p points will all fall on $a, b, c, \dots$ in a given order is $M(a.b.c\dots)/l^p$; and the chance that they will so fall in any order is $(1.2.3\dots p)M(a.b.c\dots)/l^p$.

The whole number of ways in which the $n+p-1$ points can be arranged is $(n+p-1)!$. The whole number of ways in which p points can fall on p segments (each in any order) is $(p!)(p!)$. We may then have

$$\frac{(1.2.3\dots p) M(a.b.c\dots)}{l^p} = \frac{(1.2.3\dots p)^2}{1.2.3\dots (n+p-1)}.$$

$$\text{Hence, } M(a.b.c\dots) = \frac{(1.2.3\dots p)l^p}{1.2.3\dots (n+p-1)}.$$

172. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

A circular arc, with center at one corner of a given square, is drawn through a point taken at random in the square. What is the average length of the arc within the square?

Solution by B. F. FINKEL, Ph. D., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Let $ABCD$ be the square, whose side is a , and let the coördinates of the random point P be (x, y) , A being origin, AB the x axis and AD the y axis. Then the length of the arc through the random point is $2\phi\sqrt{x^2+y^2}$, where 2ϕ is the angle between the lines drawn from the origin to the intersection of the arc with the sides of the square. Passing to polar coördinates, we have $2\phi\rho$ for the length of the arc. But $\phi = \frac{1}{4}\pi - \sec^{-1}(\rho/a)$. Hence, the length of the arc is $2\rho[\frac{1}{4}\pi - \sec^{-1}(\rho/a)] = f(\rho)$, (say). Then the average length of the arc is

$$\Delta = \frac{f(\rho_1) + f(\rho_2) + \dots + f(\rho_n)}{n}, \text{ where } n \text{ is the number of arcs,}$$

$$= \frac{\int_0^a 2\rho \frac{\pi}{4} d\rho + 2 \int_a^{\sqrt{2}a} \rho \left(\frac{\pi}{4} - \sec^{-1} \frac{\rho}{a} \right) d\rho}{\int_0^a d\rho + \int_a^{\sqrt{2}a} d\rho}$$